

Generation of three-dimensional entangled states between a single atom and a Bose-Einstein Condensate via adiabatic passage

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Abstract

We propose a robust scheme to prepare three-dimensional entanglement states between a single atom and a Bose-Einstein Condensate (BEC) via stimulated Raman adiabatic passage (STIRAP) techniques. The atomic spontaneous radiation, the cavity decay, and the fiber loss are efficiently suppressed by the engineering adiabatic passage. Our scheme is also robust to the variation of atom number in the BEC.

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I. INTRODUCTION

Quantum entanglement plays a vital role in many practical quantum information system, such as quantum teleportation [1], quantum dense coding [2], and quantum cryptography [3]. Entangled states of higher-dimensional systems are of great interest owing to the extended possibilities they provide, which including higher information density coding [5], stronger violations of local realism [6, 7], and more resilience to error [8] than two dimensional system. Over the past few years, fairish attention has been paid to implement higher-dimensional entanglement with trapped ions [9], photons [10, 11], and cavity QED [12, 13].

Atoms trapped in separated cavities connected by optical fiber is a good candidate to create distant entanglement [14–20]. The main problems in entangling atoms in these schemes are the decoherence due to leakage of photons from the cavity and fiber mode, and spontaneous radiation of the atoms [21]. By using the stimulated Raman adiabatic passage (STIRAP) [22–30], our scheme can overcome these problems.

Recently, remote entanglement between a single atom and a Bose-Einstein Condensate (BEC) was experimentally realized [31]. But the efficiency is very low due to the photon loss. In this paper, we takes both the advantages of cavity QED and STIRAP in order to create three-dimensional entanglement state between a single *Rb* atom and a *Rb* BEC at a distance. The entanglement state can be generated with highly fidelity even in the range that the cavity decay and spontaneous radiation of the atoms are comparable with the atom-cavity coupling. Our scheme is also robust to the variation of atom number in the BEC. As a result, the highly fidelity three-dimensional entanglement state of the BEC and atom can be realized base on our proposed scheme.

II. THE FUNDAMENTAL MODEL

We consider the situation describe in Fig. 1, where a single ^{87}Rb atom and a ^{87}Rb BEC are trapped in two distant double-mode optical cavities, which are connected by an optical fiber (see Fig. 1). The ^{87}Rb atomic levels and transitions are also depicted in this figure [32, 33]. The states $|g_L\rangle$, $|g_0\rangle$, $|g_R\rangle$ and $|g_a\rangle$ correspond to $|F = 1, m_F = -1\rangle$, $|F = 1, m_F = 0\rangle$, $|F = 1, m_F = 1\rangle$ of $5S_{1/2}$ and $|F = 2, m_F = 0\rangle$ of $5S_{1/2}$, while $|e_L\rangle$, $|e_0\rangle$ and $|e_R\rangle$ correspond to $|F = 1, m_F = -1\rangle$, $|F = 1, m_F = 0\rangle$ and $|F = 1, m_F = 1\rangle$ of $5P_{3/2}$.

The atomic transition $|g_a\rangle \leftrightarrow |e_0\rangle$ of atom in cavity A is driven resonantly by a π -polarized classical field with Rabi frequency Ω_A ; $|e_0\rangle_A \leftrightarrow |g_R\rangle_A$ ($|e_0\rangle_A \leftrightarrow |g_L\rangle_A$) is resonantly coupled to the cavity mode a_L (a_R) with coupling constant g_A . The atomic transition $|g_L\rangle_B \leftrightarrow |e_L\rangle_B$ ($|g_R\rangle_B \leftrightarrow |e_R\rangle_B$) of BEC in cavity B is driven resonantly by a π -polarized classical field with Rabi frequency Ω_B ; $|e_L\rangle_B \leftrightarrow |g_0\rangle_B$ ($|e_R\rangle_B \leftrightarrow |g_0\rangle_B$) is resonantly coupled to the cavity mode a_L (a_R) with coupling constant g_B . Here we consider BEC for a single excitation, the single excitation states is described by the state vectors $|G_f\rangle = \left(1/\sqrt{N}\right) \sum_{j=1}^N |g_f\rangle_j \otimes_{k=1, k \neq j}^N |g_0\rangle_j$ and $|E_f\rangle = \left(1/\sqrt{N}\right) \sum_{j=1}^N |e_f\rangle_j \otimes_{k=1, k \neq j}^N |g_0\rangle_j$ ($f = L, R$), where $|\dots\rangle_j$ describe the state of the j th atom in the BEC [31].

Initially, if the atom and BEC are prepared in the state $|g_a\rangle_A$ and $|G_0\rangle_B$ respectively, and the cavity mode is in the vacuum state. In the rotating wave approximation, the interaction Hamiltonian of the BEC-cavity system can be written as (setting $\hbar = 1$) [34]

$$H_{ac} = \sum_{k=L, R} (\Omega_A(t) |e_0\rangle_A \langle g_a| + g_A(t) a_A |e_0\rangle_A \langle g_k| + \sqrt{N} \Omega_B(t) |E_k\rangle_B \langle G_k| + \sqrt{N} g_B(t) a_B |E_k\rangle_B \langle G_0| + H.c.), \quad (1)$$

In the short fibre limit, the coupling between the cavity fields and the fiber modes can be written as the interaction Hamiltonian [15, 18, 19]

$$H_{cf} = \sum_{k=L, R} \nu_k [b_k (a_{A,k}^+ + a_{B,k}^+) + H.c.], \quad (2)$$

In the interaction picture the total Hamiltonian now becomes

$$H_I = H_{ac} + H_{cf}. \quad (3)$$

III. GENERATION OF THE THREE-DIMENSIONAL ENTANGLEMENT STATE

In this section, we begin to investigate the generation of the three-dimensional entangled state in detail. The time evolution of the whole system state is governed by the Schrödinger equation

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H_I |\psi(t)\rangle. \quad (4)$$

N_e commutes with H_I so that the excitation number is conserved during the evolution. The subspace with $N_e = 1$ is spanned by the state vectors [35]

$$\begin{aligned} |\phi_1\rangle &= |g_a\rangle_A |G_0\rangle_B |0000\rangle_c |00\rangle_f, \\ |\phi_2\rangle &= |e_0\rangle_A |G_0\rangle_B |0000\rangle_c |00\rangle_f, \\ |\phi_3\rangle &= |g_L\rangle_A |G_0\rangle_B |1000\rangle_c |00\rangle_f, \\ |\phi_4\rangle &= |g_R\rangle_A |G_0\rangle_B |0100\rangle_c |00\rangle_f, \\ |\phi_5\rangle &= |g_L\rangle_A |G_0\rangle_B |0000\rangle_c |10\rangle_f, \\ |\phi_6\rangle &= |g_R\rangle_A |G_0\rangle_B |0000\rangle_c |01\rangle_f, \\ |\phi_7\rangle &= |g_L\rangle_A |G_0\rangle_B |0010\rangle_c |00\rangle_f, \\ |\phi_8\rangle &= |g_R\rangle_A |G_0\rangle_B |0001\rangle_c |00\rangle_f, \\ |\phi_9\rangle &= |g_L\rangle_A |E_R\rangle_B |0000\rangle_c |00\rangle_f, \\ |\phi_{10}\rangle &= |g_R\rangle_A |E_L\rangle_B |0000\rangle_c |00\rangle_f, \\ |\phi_{11}\rangle &= |g_L\rangle_A |G_R\rangle_B |0000\rangle_c |00\rangle_f, \\ |\phi_{12}\rangle &= |g_R\rangle_A |G_L\rangle_B |0000\rangle_c |00\rangle_f, \end{aligned} \quad (5)$$

where $|n_{AL}, n_{AR}, n_{BL}, n_{BR}\rangle_c$ denotes the field state with n_{Ai} ($i = L, R$) photons in the i polarized mode of cavity A , n_{Bi} in the i polarized mode of cavity B , and $|n_L, n_R\rangle_f$ represents n_i photons in i polarized mode of the fiber. The Hamiltonian H_I has the following dark state:

$$\begin{aligned} |D(t)\rangle &= K \{ 2g_A \Omega_B(t) |\phi_1\rangle - \Omega_A(t) \Omega_B(t) [|\phi_3\rangle + |\phi_4\rangle - |\phi_7\rangle - |\phi_8\rangle] \\ &\quad - g_B(t) \Omega_A(t) [|\phi_{11}\rangle + |\phi_{12}\rangle] \}, \end{aligned} \quad (6)$$

which is the eigenstate of the Hamiltonian corresponding to zero eigenvalue. Here and in the following g_i, Ω_i are real, and $K^{-2} = N(g_A^2 \Omega_B^2 + 4\Omega_A^2 \Omega_B^2 + 2g_B^2 \Omega_A^2)$. Under the condition

$$g_A(t), g_B(t) \gg \Omega_A(t), \Omega_B(t), \quad (7)$$

we have

$$|D(t)\rangle \sim g_A(t)\Omega_B(t)|\phi_1\rangle - g_B(t)\Omega_A(t)[|\phi_{11}\rangle + |\phi_{12}\rangle], \quad (8)$$

Suppose the initial state of the system is $|\phi_1\rangle$, if we design pulse shapes such that

$$\begin{aligned} \lim_{t \rightarrow -\infty} \frac{g_B(t)\Omega_A(t)}{g_A(t)\Omega_B(t)} &= 0, \\ \lim_{t \rightarrow +\infty} \frac{g_A(t)\Omega_B(t)}{g_A(t)\Omega_B(t)} &= \frac{1}{2}, \end{aligned} \quad (9)$$

we can adiabatically transfer the initial state $|\phi_1\rangle$ to a superposition of $|\phi_1\rangle$, $|\phi_{11}\rangle$ and $|\phi_{12}\rangle$, i.e., $1/\sqrt{3}(|g_a\rangle_A |G_0\rangle_B - |g_L\rangle_A |G_R\rangle_B - |g_R\rangle_A |G_L\rangle_B) |0000\rangle_c |00\rangle_f$, which is a product state of the three-dimensional atom-BEC entangled state, the cavity mode state, and the fiber mode state.

The pulse shapes and sequence can be designed by an appropriate choice of the parameters. The coupling rates are chosen such that $g_A = g_B = g$, $\nu_L = \nu_R = \nu = 100g$, $N = 10^4$, laser Rabi frequencies are chosen as $\Omega_A(t) = \Omega_0 \exp[-(t-t_0)^2/200\tau^2]$ and $\Omega_B(t) = \Omega_0 \exp[-t^2/200\tau^2] + \frac{\Omega_0}{2} \exp[-(t-t_0)^2/200\tau^2]$, with $t_0 = 20\tau$ being the delay between pulses [36]. With this choice, conditions (5) and (6). Figure 2 shows the simulation results of the entanglement generation process, where we choose $g = 5\Omega_0$, $\tau = \Omega_0^{-1}$. The Rabi frequencies of $\Omega_A(t)$, $\Omega_B(t)$ are shown in Fig. 2(a). Fig. 2(b) and 2(c) shows the time evolution of populations. In Fig. 2(b) P_1 , P_{11} , and P_{12} denote the populations of the states $|\phi_1\rangle$, $|\phi_{11}\rangle$, and $|\phi_{12}\rangle$. Fig.2 (c) show the time evolution of populations of the states $\{|\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle, |\phi_6\rangle, |\phi_7\rangle, |\phi_8\rangle, |\phi_9\rangle, |\phi_{10}\rangle\}$, which are almost zero during the whole dynamics. Finally P_1 , P_{11} , and P_{12} arrive at $1/3$, which means the successful generation of the 3-dimensional entangled state. Figure 2(d) shows the error probability defined by [28]:

$$P_e(t) = 1 - |\langle D(t) | \varphi_s(t) \rangle|^2, \quad (10)$$

here $|\varphi_s(t)\rangle$ is the state obtained by numerical simulation of Hamiltonian (3) and $|D(t)\rangle$ is the dark state defined by Eq. (6). From the Fig. 2(a)-(d) we conclude that we can prepare the 3D entanglement state between single atom and a BEC with high success probability.

IV. EFFECTS OF SPONTANEOUS EMISSION AND PHOTON LEAKAGE

To evaluate the performance of our scheme, we now consider the dissipative processes due to spontaneous decay of the atoms from the excited states and the decay of cavity. We

assess the effects through the numerical integration of the master equation for the system in the Lindblad form. The master equation for the density matrix of whole system can be expressed as [20]

$$\begin{aligned}
\frac{d\rho}{dt} = & -i[H_I, \rho] - \sum_{k=L, R} \left[\frac{\kappa_{fk}}{2} (b_k^\dagger b_k \rho - 2b_k \rho b_k^\dagger + b_k^\dagger b_k \rho) \right. \\
& - \sum_{i=A, B} \frac{\kappa_{ik}}{2} (a_{ik}^\dagger a_{ik} \rho - 2a_{ik}^\dagger \rho a_{ik} + \rho a_{ik}^\dagger a_{ik}) \Big] \\
& - \sum_{j=a, L, R} \frac{\gamma_{Aa}^{ej}}{2} \left(\sigma_{e_0 E_0}^A \rho - 2\sigma_{g_j e_0}^A \rho \sigma_{e_0 g_j}^A + \rho \sigma_{e_0 e_0}^A \right) \\
& - \sum_{k=L, R} \sum_{j=k, 0} \frac{\gamma_{Ba}^{ekj}}{2} \left(\sigma_{E_k E_k}^B \rho - 2\sigma_{G_j E_k}^B \rho \sigma_{E_k G_j}^B + \rho \sigma_{E_k E_k}^B \right), \tag{11}
\end{aligned}$$

where γ_{Aa}^{ej} and γ_{Ba}^{ekj} denote the spontaneous decay rates from state $|g_0\rangle_A$ to $|g_j\rangle_A$ and $|E_k\rangle_B$ to $|G_j\rangle_B$, respectively; κ_{ik} and κ_{fk} denote the decay rates of cavity fields and fiber modes, respectively; $\sigma_{mn}^i = |m\rangle_i \langle n|$ ($m, n = E_0, E_k, G_j$) are the usual Pauli matrices. Starting with the initial density matrix $|\phi_1\rangle \langle \phi_1|$, by solving numerically Eq. (11) in the subspace spanned by the vectors (5) and $|\phi_{13}\rangle = |g_L\rangle_A |G_0\rangle_B |0000\rangle_c |00\rangle_f$, $|\phi_{14}\rangle = |g_R\rangle_A |G_0\rangle_B |0000\rangle_c |00\rangle_f$. We plot the fidelity of the entanglement state as a function of the atom decay rate γ ($\gamma = \sum_{j=a, g_l, g_r} \gamma_{Aa}^{ej} = \sum_{j=g_l, g_0} \gamma_{Ba}^{ekj} = \sum_{j=g_r, g_0} \gamma_{Ba}^{ekj}$) in Fig. 3. In the calculation, for simplicity we choose $\gamma_{Aa}^{ea} = \gamma_{Aa}^{ek} = \gamma/3$ ($k = l, r$), $\gamma_{Ba}^{ekg_0} = \gamma_{Ba}^{ekg_k} = \gamma/2$, $\kappa_{Ak} = \kappa_{Bk} = \kappa_{fk} = \kappa$, the other parameters same as in Fig. 2, and $\gamma = 0, 0.2g, 0.4g, 0.6g, 0.8g, 1.0g$ (from the top to the bottom). From the Fig. 3. we can see that the entanglement state can be generated with highly fidelity even in the range of $\gamma, \kappa \sim g$.

V. DISCUSSION AND CONCLUSION

It is necessary to briefly discuss the experimental feasibility of our scheme. Trapping ^{87}Rb BEC in cavity QED has been realized in recently experiment [34]. In this experiment, the atom number can be selected between 2,500 and 200,000 and the relevant cavity QED parameter $(g, \kappa, \gamma) = 2\pi \times (10.6, 1.3, 3.0)$ is realizable. So the condition $\gamma, \kappa < 0.4g$ can be satisfied with these system parameters for entangling the BEC and atom with fidelity larger than 98%. The BEC spatial atomic mode and cavity mode have the overlap $\mu(N) =$

$\sqrt{0.5}(1 - 0.0017N^{0.34})$, so the coupling strength $g_B(t)$ will decrease with increasing atom number N . We can increase the $\Omega_B(t)$ accordingly to compensate this. The fidelity versus the atom number N of the BEC is plotted in Fig. 4 with the parameters $\gamma = \kappa = 0.4g$, and the other parameters same as in Fig. 2.

In summary, based on the STIRAP techniques, we propose a scheme to prepare three-dimensional entanglement state between a BEC and a atom. In this scheme, the atomic spontaneous decay and photon leakage can be efficiently suppressed, since the populations of the excited states of atoms and cavity (fiber) modes are almost zero in the whole process. We also show that this scheme is highly stable to the variation of atom number in the BEC. Recently, strong atom-field coupling for Bose-Einstein condensates in an optical cavity on a chip [37] and strong coupling between distant photonic nanocavities [38] have been experimentally realized. So our scheme is considered as a promising scheme for realizing entanglement between BEC and atom.

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Figure caption (without color)

Fig. 1 A single ^{87}Rb atom and a ^{87}Rb BEC are trapped in two distant double-mode optical cavities, which are connected by an optical fiber. The states $|g_L\rangle$, $|g_0\rangle$, $|g_R\rangle$ and $|g_a\rangle$ correspond to $|F=1, m_F=-1\rangle$, $|F=1, m_F=0\rangle$, $|F=1, m_F=1\rangle$ of $5S_{1/2}$ and $|F=2, m_F=0\rangle$ of $5S_{1/2}$, while $|e_L\rangle$, $|e_0\rangle$ and $|e_R\rangle$ correspond to $|F=1, m_F=-1\rangle$, $|F=1, m_F=0\rangle$ and $|F=1, m_F=1\rangle$ of $5P_{3/2}$. The atomic transition $|g_a\rangle \leftrightarrow |e_0\rangle$ of atom in cavity A is driven resonantly by a π -polarized classical field with Rabi frequency Ω_A ; $|e_0\rangle_A \leftrightarrow |g_R\rangle_A$ ($|e_0\rangle_A \leftrightarrow |g_L\rangle_A$) is resonantly coupled to the cavity mode a_L (a_R) with coupling constant g_A . The atomic transition $|g_L\rangle_B \leftrightarrow |e_L\rangle_B$ ($|g_R\rangle_B \leftrightarrow |e_R\rangle_B$) of BEC in cavity B is driven resonantly by a π -polarized classical field with Rabi frequency Ω_B ; $|e_L\rangle_B \leftrightarrow |g_0\rangle_B$ ($|e_R\rangle_B \leftrightarrow |g_0\rangle_B$) is resonantly coupled to the cavity mode a_L (a_R) with coupling constant g_B .

Fig. 2 The numerical simulation of Hamiltonian (3) in the entanglement generation process, where we choose $g = 5\Omega_0$, $\tau = \Omega_0^{-1}$. Figure 2(a): the Rabi frequency of $\Omega_A(t)$, $\Omega_B(t)$. Fig. 2(b): the time evolution of populations of the states $|\phi_1\rangle$, $|\phi_{11}\rangle$, and $|\phi_{12}\rangle$ is denoted by P_1 , P_{11} , and P_{12} respectively. Fig.2 (c): time evolution of populations of the states $\{|\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle, |\phi_6\rangle, |\phi_7\rangle, |\phi_8\rangle, |\phi_9\rangle, |\phi_{10}\rangle\}$ are almost zero during the whole dynamics. Figure 2(d): error probability $P_e(t)$ defined by Eq. (6).

Fig. 3 Fidelity of the entanglement state as a function of the atom decay rate γ (obtained by numerical simulation of master equation (8)) and the atom decay rate $\gamma = 0, 0.2g, 0.4g, 0.6g, 0.8g, 1.0g$ (from the top to the bottom).

Fig. 4 Fidelity vs the atom number N of the BEC with the parameters $\gamma = \kappa = 0.4g$, and the other parameters same as in Fig. 2.





